Económica. La Plata. Vol. XLIV. Nro. 1-2. Año 1998

DISEQUILIBRIUM ECONOMICS: TOOLS FROM OPEN SYSTEMS THEORY*

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Introduction

In recent years several authors, discussing the logical structure of neoclassical economics, emphasized the influence of Newtonian mechanics on its early development (Georgescu-Roegen, 1971). The main support for this argument comes from the fact that one of the core elements of the so called neoclassical thought, marginal analysis, is deterministic and equilibrium-oriented, qualities associated with philosophical mechanicism (Mirowski, 1989).

Despite those arguments, the philosophical influence on early neoclassical authors can be also traced back to another branch of physics, *thermodynamics*, with its emphasis on the convergence of systems to a static state. Thermodynamics, as known by the economists of 1870, was a phenomenological theory, concerned with the description of the behavior of macrovariables (states) in systems with many degrees of freedom. The corresponding formalism derived the steady state of a closed thermodynamic system as the result of a balance of the flows (of energy and matter) in it. Considering one of main formal representations in neoclassical economics, Walras' tâtonnement, the system is closed to external influences, and the only 'flow' is the change of prices, which tended to an equilibrium reached only when that flow is null (Arrow-Hahn, 1971).

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* This is a renewed version of a paper that we were invited to present at the XXX Meeting of the Asociación Argentina de Economía Política (1995), and finally was presented at the XIV Latin American Meeting of the Econometric Society (Río de Janeiro, 1996). We want to give our thanks to the useful comments of Prof. Julio H. Olivera, Prof. Daniel Heymann and an anonymous referee. Errors, of course, are of our own responsability.

JEL Classification: B41

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The influence of thermodynamics became more noticeable after the 1930s, thanks to the influence of Paul Samuelson (*comparative statics* was a customary method of analysis in phenomenological thermodynamics) and, on the other hand, to the development of probabilistic-oriented econometrics (statistical analysis of systems was another feature of thermodynamics since its reformulation initiated by L. Boltzmann). In both cases the influence was decisive in the selection of economic formalizations.

Given the importance of formalisms drawn from thermodynamics to economic theory, it seems interesting to explore the possibility of applying the latest devolopments in the realm of the so called Open Systems Theory (O.S.T.) to the modellization of economic phenomena. Formalisms in this set of extensions of thermodynamics to non-equilibrium open systems are concerned with the dynamics of systems with many components, subject to external shocks. The high degree of generality of these formalisms make them suitable modeling tools for phenomena in different realms. Economics, in particular, needs formalisms to handle situations in which several linked components (agents, firms, sectors, etc.) are subject to perturbations which throw the system out of equilibrium. These situations are theoretically relevant in areas that range from Development Economics to International Economics. The complexity of the interactions and the non-stationarity of the rules of behavior makes those cases candidates to be studied using methods from O.S.T..

In the next section, we present a succinct overview of the several (and sometimes conflicting) emerging disciplines we unite under the common umbrella of O.S.T.: synergetics, dissipative structures, etc. As examples, in section 3 a walrasian dynamics (tâtonnement) will be analyzed as a self-organízed system, and in section 4 we will show how economic development can result from non-linear interactions among several sectors.

Open Systems Theory

It can be said -with extreme generality- that thermodynamics studies the reactions of a system to changes in its environment. When the consideration shifts to *open* systems (which exhange continuously energy and mass with their environments) the main phenomena of concern are related to the unpredictability and the complexity of the systems.

There are three kinds of thermodynamical systems, each one associated with different long term conditions:

- **Isolated systems**, which show no exchanges with their environments. They always reach a state of equilibrium.
- **Closed systems**, which only exchange energy with their environments. They reach a regime near equilibrium.
- **Open systems**. Their structures do not remain fixed in time.

A structure is a characterization of the behavior of a system in a region of the space of its parameters. This space can be partitioned in stable and unstable regions, according the reaction of the solutions to a slight change in parameters (Guckenheimer-Holmes, 1983). For isolated and closed systems remains valid Boltzmann's principle, which states that changes go always from less to more probable states, being equilibrium the most probable state of such systems. This principle, which explains why both kinds of systems show convergence to equilibrium, is no longer valid for open systems, since any change of parameters is followed by changes in the probability distribution on states.

A common feature of large open systems showing weak interactions among their components, is the existence of *macrostructures*, each one showing a great number of components generating a colective behavior. When macrostructures grow, the system becomes more stable, since it requires a greater effort to rearrange its (bigger) components. On the other hand, with a great number of macrostructures the possibility of unstable behavior increases, since the spectrum of feasible reactions to external perturbations widens. Therefore, there exist tresholds of structural stability determined by the size of macrostructures (Varian, 1981).

There are several approaches to the analysis of open systems. Their common features are related to the utilization of non-linear systems of differential (or difference) equations for their representation. The mathematical analysis of non-linearities shows that given the values of parameters, there can exist several attractors in the state space, to which the temporal path of the system may converge according to the position of the initial state. The partition of the state space in attraction basins varies with changes in parameters, generating *bifurcations*, where abrupt changes of the landscape lead to abrupt changes in behavior.

The main properties of O.S.T. methodologies, which we consider important for Economics, are the following :

- non-linear relations can be represented
- structures change according to changes of parameters
- changes are irreversible ("history" matters)

Given the characterization above, the *theory of dissipative structures* studies the path of bifurcations (and their associated macrostructures) that a system subject to random perturbations follows (Haken, 1988). Synergetics, instead, considers phenomena of *self-organization*, in which eigenvalues of perturbations around a critical parameter are classified as stable or unstable. According to the relative values of the unstable parameters the system reorganizes attaining a new solution path or returns to the original one (Prigogine, 1989).

A Self-organized Tâtonnement

The first area of influence of classical thermodynamics in economic theory was the representation of market economics. In particular, it was assumed that the market under consideration remained closed to external influences. If the state of the market is described completelly by the vector of prices, a story was needed to explain why markets were so often in equilibrium, that is, why demand and supply seemed to match so well. Leon Walras presented a dynamics of price adjustments tending to equilibrium. In his presentation, based in an analogy with an auction, no transactions were realized out of equilibrium. Despite several formal problems in Walras'presentation, auction-like models of price adjustement became pervasive, under the general title of **tâtonnement dynamics**. The modern versions concentrate on the change of prices as a function of the demand excesses: prices of commodities with excess demand grow while prices of goods with excess supply fall. Equilibrium is therefore a state in which excesses are null. A great deal of effort was devoted to derive conditions for the existence, stability and uniqueness of equilibrium. While examples in which tâtonnement did not converge to equilibrium were constructed, it is still considered a procedure that reflects key elements of the dynamic behavior of prices.

The assumption of a market closed to external influences can be relaxed to allow a variability of the parameters of the excess demand. More precisely, the price dynamics can be expressed as follows :

- $dp = Z(\alpha, p)$

where p is the system of prices in the unitary positive ball S^+ and $Z(\alpha, p)$ is a non-linear excess demand function[1]. The parameter α, represents the possibility of changing preferences or endowments (or both). A rather obvious geometrical representation of this system is given by a vectorial field on S^+.

Implicit here is the assumption that the economy is a pure exchange one, where the exogenous variables are the characteristics of the agents, i.e. their preferences and endowments. To assume that the market is open implies to say that the characteristics of the agents may change. Reasons for such changes can be income shocks (which increment or decrement the endowments) or changes of preferences (due either to internal reasons or to external influences like publicity).

Let $p^*(\alpha, t)$ be the equilibrium price. The characteristics of the agents change, that is, the equilibrium suffers a perturbation. The evolution of a perturbation can be expressed:

- $p(\alpha, t) = p^*(\alpha, t) + w(\alpha, t)$

$w(\alpha, t)$ is the path of the perturbation to the equilibrium state. Taking derivatives respect time it can be seen that ($\partial p^*(\alpha, t) = 0$):

- $\partial p(\alpha, t) = \partial w(\alpha, t)$

In a neighborhood we can take a linear approximation:

[1] The excess demand function should be continuous, homogeneous of degree zero and it has to verify Walras Law: $p.Z = 0$.

- $\partial w(\alpha, t) = k \bullet w(\alpha, t)$

and therefore:

- $w(\alpha, t) = k^* \bullet e^{kt}$, where $k^* = w(\alpha, 0)$

Given that $\partial p = \partial w$, the eigenvalues of the linear approximation are the solutions to the following equation:

- $k \bullet w = \partial Z / \partial p \bullet w$

Each solution k_i determines a direction of motion and $Re(k_i)$ the magnitude of that motion. Given the linear independence of the eigenvectors we can represent the perturbation as follows:

- $p(\alpha, t) = p^*(\alpha, t) + \Sigma_i k^*_i \bullet e^{kit}$

This expression will be identical to $p^*(\alpha, t)$ when $t \rightarrow \infty$, for values of α such that for all i, $Re(k_i) << 0$.

But if α becomes critical (there exists an i such that $Re(k_i) = 0$) we can make a bipartition of the set of eigenvalues. A class will consist in the stable eigenvalues : k_i^s, such that $Re(k_i^s) < 0$. The other class will include the unstable eigenvalues: k_i^u, with $Re(k_i^u) \geq 0$. So we have:

- $p(\alpha, t) = p^*(\alpha, t) + \Sigma_i k^*{}_i^s \bullet e^{kis} + \Sigma_i k^*{}_i^u \bullet e^{kiut}$

The $\{e^{kist}\}$ and $\{e^{kist}\}$ are called the stable and unstable *modes* of behavior of the system. The time derivative of the modes or *relaxation amplitude* shows that:

- $\partial e^{kist} / \partial t = k_i^s \bullet e^{kist}$
- $\partial^{ekiut} / \partial t = k_i^u \bullet e^{kiut}$

It is clear that $\partial e^{kist} / \partial t << \partial e^{kiut} / \partial t$. The stable modes relax quicky and so they are able to follow the slow relaxation of the unstable modes. This process is called *enslaving* of the stable modes. An *invariant manifold* can be defined (Prigogine, 1989):

- $e^{kist} = e^{kis\bullet \exp(kiut)}$

Therefore the system losses degrees of freedom and its whole behavior, when a parameter becomes critical, is determined by the unstable modes. An equilibrium becomes unstable at a change of parameter and the tâtonnement self-organizes taken the system to a new state[2]. To show more intuitively how this works we can take an exchange economy with two commodities.

By Walras law we can take the excess demand of commodity 1, Z_1 (α; p_1/p_2, 1) (prices can be normalized because the excess demand must be homogeneous of degree zero). Any equilibrium will be of the form (p^*_1/p^*_2, 1), and it will be stable if $\partial Z_1(\alpha; p^*_1/ p^*_2, 1) < 0$.

Any perturbation to the equilibrium will be of the form:

- $p_1(t) / p_2(t) = p^*_1 / p^*_2 + k^*\bullet e^{\partial Z1(\alpha;\ p^*1/\ p^*2,\ 1)\ \bullet\ t}$, where k^* is the value of the perturbation in $t = 0$

From the discussion above it is clear that a self-organization will arise when $\partial Z_1(\alpha; p^*_1/ p^*_2, 1)$ changes sign after a change in α. We represent this fact in caption I, respect an equilibrium p^*_1/ p^*_2 (we can think it as a change in preferences that transforms good 1 from normal to Giffen):

- $\alpha = \alpha_1$, $\partial Z_1(\alpha; p^*_1/ p^*_2, 1)$ is negative, so any perturbation dies after a short time
- $\alpha = \alpha_2$, $\partial Z_1(\alpha; p^*_1/ p^*_2, 1)$ is null, so the result is that any perturbation does not increase nor decrease in time

[2] It can be a cycle, as shown by Scarf (1960).

- $\alpha = \alpha_3$, $\partial Z_1(\alpha; p^*_1/p^*_2, 1)$ is positive, so any perturbation grows and the system does not return to p^*_1/p^*_2 [3].

This very simple example shows how a change in a "given" to the standard GE model can trigger a global change of behavior of the economy. This is related to the concept of *regular economies*, introduced by Debreu (1970). Obviously a value of α such that in an equilibrium $\partial Z_1(\alpha; p^*_1/p^*_2, 1) = 0$ makes the economy not regular. An interesting result is that almost every exchange economy (with fixed parameters) is regular, so the cases in which self-organization arises are not generic. But the point of this example is to show how an economy in continuous transformation can naturally pass through such a non-regular state despite its non-genericity. This is, maybe, a pointer towards a theory of equilibrium selection (Mas-Colell et al. 1995).

Inter-sectorial Dynamics and Economic Development

An open tâtonnement economy may seem a bit farfetched. This is not the case of economic development, which has not yet a satisfactory representation. In the following model, which tries to formalize this phenomenon, the variables and parameters are:

- n : number of sectors in the economy
- σ_i : degree of development of sector i
- O^p_i : potential output of sector i
- O^a_i : actual output of sector i
- D_i : demand of products of sector i

It is assumed that there are intersectorial multiplier effects, that is that the production of a sector influences its own future growth. If the potential output of all sectors grow at a rate τ we have the following relation:

- $\partial \log \sigma = \partial \log O^a_i - \tau \quad \forall i$

[3] Lets note that cases 2 and 3 do not verify the Index Theorem (Varian, 1981).

Production plans of each sector i react with delay to the demand of its products:

- $O_i^{at} = F_i (D_i^{t-1})$

Variations of demand of a sector i are a result of variations in the production of the other sectors j, weighted by the requirement of products of i in the production of j (k_{ij}) :

- $D_i^t - D_i^{t-1} = \sum_{i \neq j} k_{ij} \ (O_j^{at} - O_j^{a(t-1)}) + H_i^t$

where H_i is a component of exogenous variation.

The functions $\{F_i\}$ are the key elements in this model. We will make two strong assumptions about these response functions: the first one is that they are identical for all sectors in the economy. This means that -as the responses are generated by expectations and productive capabilities- all sectors are homogeneous in their knowledge of the global economic performance and also in their technical conditions. To be consistent with this assumption, the growth rate of the Potential Product τ has to be the same for all sectors. Also for consistency we will assume that stocks are not accumulated since the accurate short range forecasts of economic performance allow the sectors to meet the needs of their buyers.

The second strong assumption about the response functions is that the underlying costs are not convex when aggregate demands are small. This may obey to indivisibilities which at large demands become smoothed out. As a consequence, in low ranges, responses will be non-linear. At larger demands, instead, the smoothing out of indivisibilities allow a monotonicity in responses. Moreover, we assume that in such a situation expectations will be of growth and responses will be monotonic.

In order to represent these characteristics we have to introduce a "toy model". As usual with simulation models, genericity cannot be ascertained : experiments can be performed for a finite number of possible values of the parameters. Despite this limitation, the usefulness of working with toy models is obvious (Ruelle, 1988). In our case, we choose to give the function F_i the following characterization, exhibiting the features discussed above:

$$F_i(x) = \begin{cases} O_{pi} & \text{if } x \geq O_{pi} / \alpha \\ \alpha.x & \text{if } x \in [\,\delta\,.O_{pi} / \alpha\,,\, O_{pi} / \alpha] \\ \beta.\sin(x) + \mu & \text{otherwise} \end{cases}$$

This model, a kind of coupled partial difference equations (Brock, 1991), can not be handled analytically. Therefore it has to be simulated giving values to its parameters, obtaining through a very simple numerical integration the desired trajectories. For this model we chose the following parameter values:

- n : the number of sectors
- $\alpha, \beta, \delta, \mu$
- $\{k_{ij}\}_{i,j} = 1..n$
- $O_{Pi}^{\ 0}$, $D_i^{\ -2}$, $O_{Ai}^{\ -2}$, $O_{Ai}^{\ -1}$, $H_i^{\ -1}$

With these data at hand the evolution of $\{\sigma_i^{\ t}\}_{i=1..5}$ can be followed for t = 0..T.

To bound the number of simulations, several parameters will be considered constant:

- $n = 5$
- $\tau = 0.05$
- β = random in [0,1]
- $\delta = 0.7$
- $\mu = 0.5.\ \max_i O_{Pi}$
- $\|k_{ij}\|$: matrix of random numbers in [0,1], normalized by rows and columns
- $T = 50$

Given these constraints, the following numerical experiments represent, each one, a family of interesting analogous behaviors (see Captions):

- For i = 1..5 $O_{Ai}^{-1} = O_{Ai}^{-2} = 70$; $D_i^{-2} = 70$; $O_{Pi}^{-2} = 100$; $H_i^{-1} = 0$; $\alpha = 1.1$
- For i = 1..5 $O_{Ai}^{-1} = O_{Ai}^{-2} = 80$; $D_i^{-2} = 80$; $O_{Pi}^{-2} = 100$; $H_i^{-1} = 1$; $\alpha = 1.5$
- For i = 2..5 $O_{Ai}^{-1} = O_{Ai}^{-2} = 60$; $D_i^{-2} = 60$; $O_{Pi}^{-2} = 100$; $H_i^{-1} = 10$; $\alpha = 1.1$ and $O_{A1}^{-1} = O_{A1}^{-2} = 80$; $D_1^{-2} = 80$; $O_{P1}^{-2} = 100$; $H_1^{-1} = 10$; $\alpha = 1.1$

Experiment 3 shows that the economy attains, in the beginning, a joint high degree of development. Then sectoral disparities appear and the system declines. The initially less developed sectors are the first to decrease, followed by the initially more advanced one. It can be argued that an initial disequilibrium of the system (a demand shock in the beginning) is not enough to stabilize it at a high degree of development, and that further positive shocks are necessary to maintain the degree of development attained. These results are reminiscent of the *hypothesis of unbalanced growth*, where the existence of disequilibria is a condition for the advance of the economy (London-Tohmé, 1993a; Hirschman, 1975). The initial disparities are not a requirement for this behavior. Experiment 1 shows that a homogenously developed economy can grow (or remain stable) in the first stages and without demand shocks it declines to degrees below the initial values.

The explicative importance of α decreases when the system begins with all the sectors developed at a higher degree. Then, a very low initial demand shock is enough for incrementing the degree of development (experiment 2).

This conclusion is consistent with the hypothesis of the Big-Push (Higgins, 1959): once the economy surpasses a critical threshold it attains, almost with certainity, a high degree of development[4].

It would be more interesting to obtain permanent desequilibrium, a case in which the degrees of development where different for each sector. This result obtains considering different functions for each sector, all having at least a *non-linear* section. If the compound $F \equiv (F_1,\ldots,F_n)$ has a cuadratic form there is a real possibility that the system can attain, for certain initial values, a chaotic behavior (Kelsey, 1988).

[4] For a rationale and a deeper analysis of these results see London-Tohmé (1993b).

Conclusions

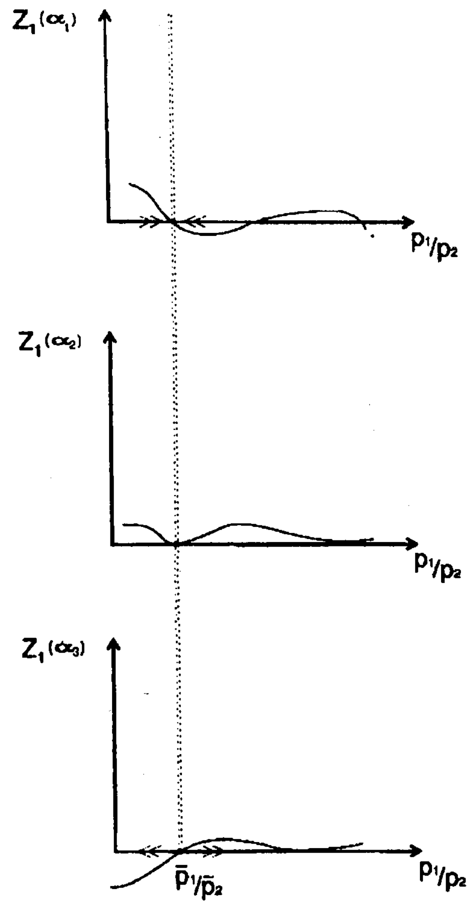
Analytical methods developed in Open Systems Theory can be useful as modelling tools in economics, allowing to incorporate in an elegant way the following notions:

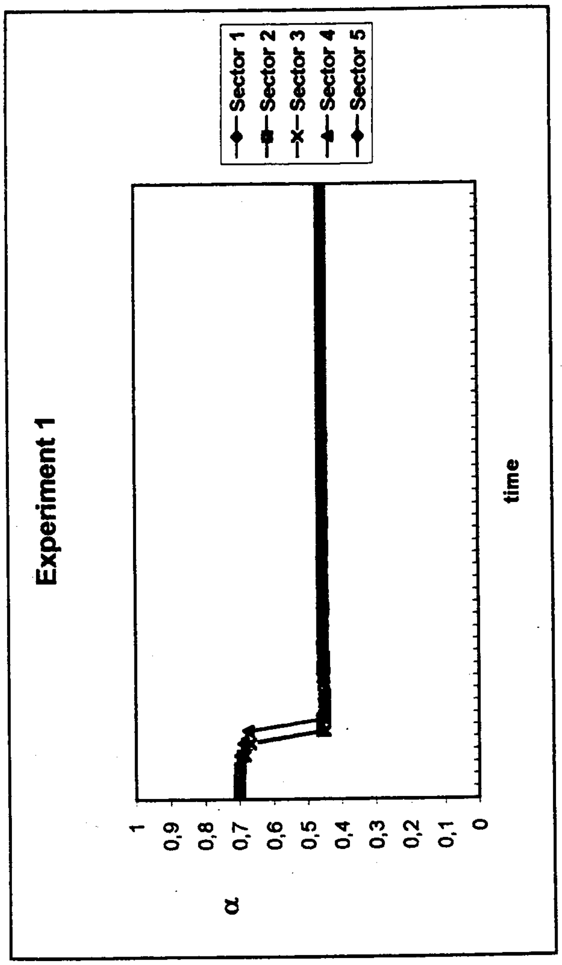
- economic disequilibria
- historical processes
- uncertainity as a fundamental factor in economic behavior

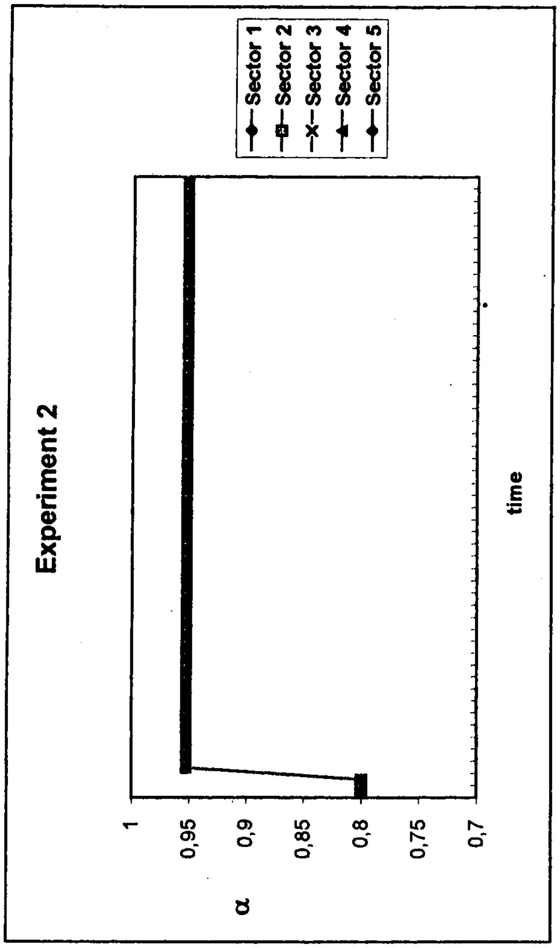
To elaborate : the three factors above are all related to the fact that structural changes occur in non-linear systems where the parameters change. The history of the process is an enumeration of metastable equilibria (stable for ranges of values of parameters), which is not cleary distinguishable from disequilibria. Moreover, the transition from a structure to other obey to the action of uncertain factors on otherwise deterministic processes.

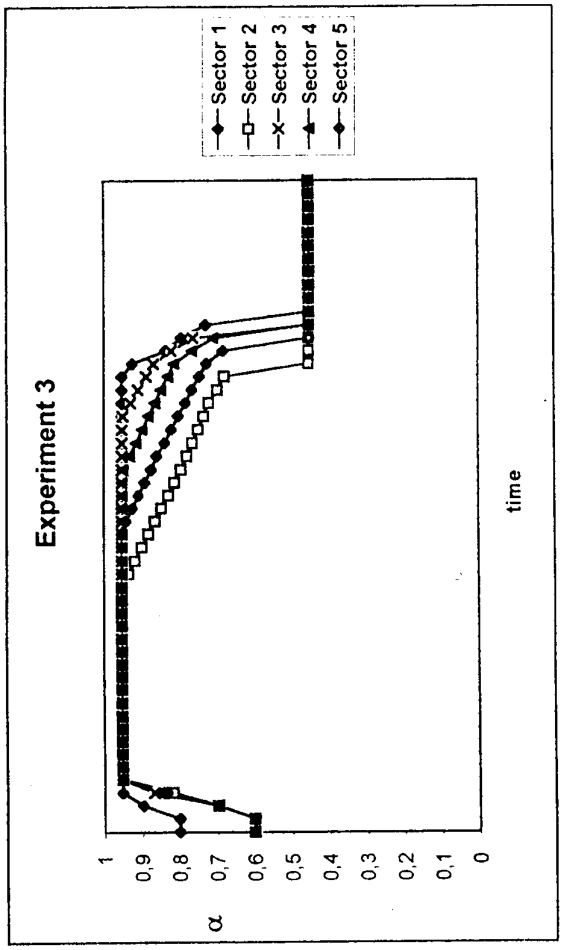
The reason for the applicability of thermodynamical formalisms to Economics can be found in the common features of economies and physical systems with a great number of components. The analogy works better in the case of open systems, with their ever changing and unstable behaviors. Of course, substantial differences exists and the analogy should not be pushed beyond certain limits. The generality of O.S.T. makes it easier for the economist (as it has been for other scientists) to apply its tools to the analysis of phenomena not directly related to thermodynamics. The examples we presented can be extended to study complex phenomena like hyperinflations, financial crashes, etc (see Becker (1994) for a deep discussion in the same vein as this paper). Economic analysis of those situations could benefit from the introduction of these new tools.

Caption I: Self - Organized Tatonnement




Experiment 1
Sector 1
Sector 2
Sector 3
Sector 4
Sector 5
α
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
time


Experiment 2
Sector 1
Sector 2
Sector 3
Sector 4
Sector 5
α
1
0,95
0,9
0,85
0,8
0,75
0,7
time


Experiment 3
Sector 1
Sector 2
Sector 3
Sector 4
Sector 5
α
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
time

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DISEQUILIBRIUM ECONOMICS: TOOLS FROM OPEN SYSTEMS THEORY

SUMMARY

The analogies between economic systems and thermodynamic systems are both obvious and misleading. Even if important thermodynamic notions lack of counterparts in Economics, the transference of formal devices became a feasible goal. Procedures like comparative statics were imported directly from thermodynamics, making possible to relate equilibria in economic systems to properties of their environments.

Continuing that tradition we claim that it would be good for Economics to borrow formalisms from Open Systems Theory, the not yet well defined set of extensions of Thermodynamics to the analysis of open systems. In this paper we discuss briefly the epistemological rationale for this claim and present two models that cannot be analyzed in classical terms as to show how the tools of O.S.T. can be applied.

DESEQUILIBRIO ECONOMICO: HERRAMIENTAS DESDE LA TEORIA DE LOS SISTEMAS ABIERTOS

RESUMEN

Las analogías entre sistemas económicos y sistemas termodinámicos son a la vez obvias y engañosas. Aún si importantes nociones de la termodinámica no encuentran su contraparte en Economía, la transferencia del aparato formal se convierte en un objetivo factible. Procedimientos tales como estática comparativa fueron importados directamente de la termodinámica, haciendo posible relacionar equilibrios de sistemas económicos con propiedades de su entorno.

Sería bueno, continuando con esa línea, que la economía tomase formalismos de la Teoría de Sistemas Abiertos, el aún no definido conjunto de extensiones de la Termodinámica al análisis de sistemas abiertos. En este trabajo discutimos brevemente la justificación epistemológica de tal propuesta, y presentamos dos modelos a modo de ejemplo.