Económica, La Plata, Vol.XLV, Número Especial, Año 1999

EQUILIBRIO CON RENDIMIENTOS CRECIENTES A ESCALA*

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Introduction

In general equilibrium the production possibility sets and consumer preferences are convex. The reason for this assumption resides in two problems which arise when it is not made.

On the one hand, if indifference curves in the theory of the consumer or isoquants in the theory of the producer do not have the appropriate curvature, there will be discontinuities in the demand and supply correspondences. This type of non-convexities has been adequately treated in the investigations of Aumann (1966), Schmeidler (1969), and Hildenbrand (1974), who demonstrated that if the number of consumption units and firms is large, so that everyone has a negligible influence on the economy, the latter can be approximated by one in which these curves have the appropriate curvature. Intuitively, even if each participant acts discontinuously, if he is sufficiently small relative to the economy, the jumps in his decisions will be unnoticed by the aggregate.

On the other hand the existence of economies of scale in production have more negative effects on the existence of a competitive equilibrium solution, since in that case the assumption of profit maximization is incompatible with the behavior of a price taking entrepreneur. Such a behavior would induce the firm with increasing returns to scale to expand indefinitely, so that equilibrium would be impossible.

Several solutions to this problem haven been suggested, the importance of which cannot be denied thanks to the wide empirical evidence that marks the existence of increasing returns in certain sectors. Among the extremes is the solution provided by the theory of equilibrium with monopolistic competition

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* Originalmente la presentación se realizó en castellano. Sin embargo esta es una versión más moderna de ese mismo trabajo.

** Anales de la Asociacion Argentina de Economia Politica. Facultad de Ciencias Economicas de la Universidad Nacional de Cuyo, November 1979, vol. II, 271-82.

developed by the pioneering investigation by Negishi (1960) and continued by Arrow and Hahn (1972), and the central planning solution, divorced from the consumer sovereignty usually found in general equilibrium models.

In the present investigation it will be shown that the problem of the existence of an equilibrium solution can also be resolved in a model in which one introduces government controls on the sectors which ---because of the magnitude of possible increasing returns to scale--- threaten to exploit other sectors with their monopolistic power. That is to say, sectors with increasing returns to scale will be aggregated under the generic designation of the "regulated sector". This will be understood as the aggregate of private or public enterprises which show exceptions to the convexity assumption in their production possibility sets which are of sufficient weight as to affect significantly market quantities or prices. The model will be formally presented in the next section; another section will be dedicated to the proof of existence of an equilibrium solution.

The model

One assumes the existence of m individuals designated by $i=1,\dots,m$ trading n commodities labeled with $j=1,\dots,n$. The quantities that individual i obtains from the market will be represented by the vector $x^i \varepsilon R_+^n$, the nonnegative orthant of n-dimensional Euclidean space. Following Meade (1952) and Rader (1964), those traded quantities correspond to each individual's consumption, net of own production, plus inputs needed for the production processes carried out by each unit, given the usual assumptions of convexity and non interference of one activity with another, be it either consumption or production. This device permits a simplification of the notation without sacrificing the generality of the analysis, and to center the analysis on the substantial aspects of the present essay.

The usual model of general equilibrium is included in the model here presented, if the following assumptions are made on trade induced preferences - where B_i represents the strict preference correspondence of the i -th consumer-,
C1) Continuity: The graph of the correspondence B_i, that is to say of the set $\{(x,y) | x \varepsilon B_i(y)\}$, is open in the Cartesian product $R_+^n \times R_+^n$.

C2) Irreflexivity: The point x is not in the convex hull of its image by B_i, that is to say, x is not preferred to some mix of points preferred to it.

In a pioneering article, Mas-Colell (1974) demonstrated the existence of a competitive equilibrium in a model such as that here described; note that it is not necessary to assume that consumers have preferences that are preorders, as is usual. Nevertheless, in order to simplify the following exposition and proof, we will limit ourselves to the proof of the existence of an equilibrium in the case in which the following assumptions also are satisfied.

C3) B_i is convex for all $x \in R_+^n$.

C4) The induced preference relation B_i is transitive, complete, and strictly convex.

The general case is treated in an appendix which can be requested from the author.

The characteristic which distinguishes the present investigation from its predecessors is the introduction of a sector whose production possibility set Y is not convex; its interpretation as a regulated sector has been made in the introduction.

The following assumptions will be made on the regulated sector.

P1) Y is a compact subset of R_+^n with nonempty interior.

P2) Y is a comprehensive set, i.e., if y is a point in Y, and if the inequality $0 \leq y \leq y'$ holds, then y' also is in Y.

P3) Y has a smooth boundary, i.e., for each point y in its boundary with strictly positive coordinates there is a unique normal with direction coefficients given by the vector p (y) which depends continuously on y.

The meaning of these assumptions is the following. P1) says that it is impossible to produce arbitrarily large quantities. It also says that all commodities can be produced. The latter is not so far from reality as it sounds, since it always is possible to include convex production possibility sets in Y.

P2) says that if a certain production plan y is feasible, than any plan which offers net outputs which do not exceed those of y also will be possible.

Assumption P3) is surely more farfetched. Nevertheless this assumption is not strictly needed, and is used here only to simplify the argument. It is perfectly possible to carry out the demonstration assuming that the set Y has the property that in another essay has been labeled as having "a frontier defined by a locally concave function", which in essence requires that the cone of normals to the set Y be convex at each of its boundary points.

Finally, it is assumed that there exist m continuous functions for the distribution of the proceeds of the sector with increasing returns, so that:

D1) $a^i(y)$ is positive for all $y \varepsilon Y$,

D2) $\sum_i a^i(y) = y \cdot p(y)$ for all $y \varepsilon Y$.

The first of these conditions requires that all individuals participate positively from the product of the sector of increasing returns, be it as shareholder of a private firm, or as owner of a right to part of a public enterprise. D2) requires that the whole product of the regulated sector be distributed.

Equilibrium

A solution to the model of the previous section is defined as a vector of prices $\bar{p} \varepsilon R_+^n$, a production plan $\bar{y} \varepsilon Y$, and m trading plans $\bar{x}_i \varepsilon R_+^n$ such that one has:

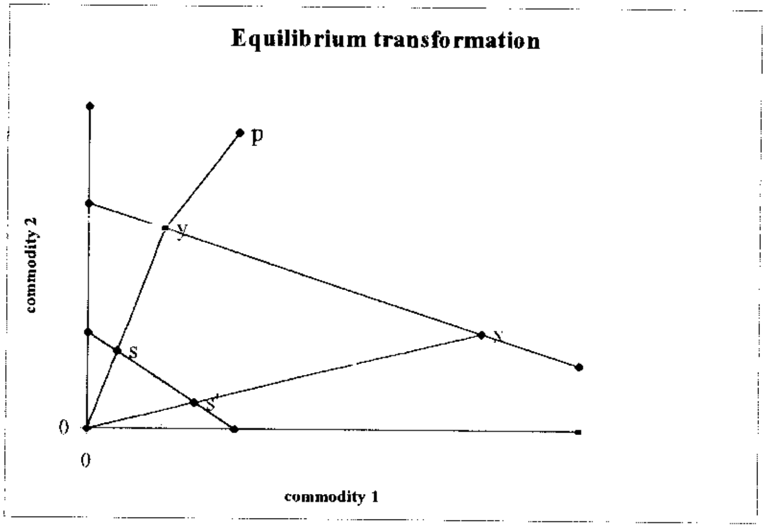
E1) Market balance: $\sum_i \bar{x}_i \leq \bar{y}$,

E2) Maximum satisfaction: $\bar{p} \times \bar{x}_i \leq a^i(\bar{y})$; and $x_i \varepsilon B_i(\bar{x}_i)$ implies $\bar{p}\ x_i > a^i(\bar{y})$,

E3) Marginal pricing: $\bar{p} = p(\bar{y})$.

The meaning of E1) is obvious. E2) requires that each individual consumer maintain its expenses within the limit imposed by its participation in the increasing returns sector, but that within this restriction there be no other trade plan preferred by him. E3) says that the price vector is normal to the production possibility set Y, so that relative prices reflect the marginal rates of substitution in the production of commodities.

Due to the assumptions on the production possibility set Y it is possible to see immediately that its boundary, in its intersection of the n-dimensional positive orthant, is homeomorphic to a simplex S of dimension n-1. Therefore it is possible to apply Mas-Colell's argument, even though for the author just mentioned the set Y reduces to just one point. Here a somewhat different construction will be presented, used by the present author in "General equilibrium and optimal taxes" (1975). See the accompanying figure, where the curve is the upper boundary of the production possibility set Y, and the segment containing the points s and s' is the one-dimensional unit simplex S.



Starting with a point s ε S, the unit simplex, define the point y as the intersection of the half-line with origin 0 through s with the boundary of Y. Take as price vector the coordinates of the normal p(y) to Y at y; this vector can be assumed to be normalized, so that it is in S, since only relative prices are relevant.

With this information it is possible to compute the income each individual receives from the sector of decreasing costs, given by the function $a^i(.)$. Thanks to assumptions C1-C2 there is a continuous demand function for each i; from the individual demands x^i, via aggregation, one can derive the collective demand $x = \sum_i x^i$, shown in the figure on the tangent to Y at y. The intersection of the half-line through x with origin 0 with the simplex S gives point s'. As a consequence of all previous arguments, s' will be a continuous function of s, with arguments and image in the same compact, convex set S. Brouwer's fixed point theorem guaranties then the existence of at least one point s ε S equal to its image s'. It is immediate that prices, production plans, and trade plans corresponding to that fixed point s satisfy the equilibrium conditions E1-E3.

Historical note

The present investigation originated during a brief visit to the University of Minnesota in March, 1976. Donald Brown had been there a week before, and had presented a version of his investigation with Geoffrey Heal on non convex production possibility sets. David Schmeidler was present, and had some doubts both on the existence and the optimality of an equilibrium solution in the model presented by Brown, which he transmitted to the present author. This explanation shows how indirectly the transmission of knowledge can sometimes be, since at the time the author was a Visiting Professor at Yale University, where Brown and Heal were developing their ideas.

Thanks to Schmeidler's question it occurred to the author to adapt the argument used for another model (1975) to this case, arriving thus at the proof of existence of an equilibrium. Regarding its optimality, somewhat later the author could present an example in which the unique equilibrium that respects the market forces is inefficient, due to the non-convexity of the production possibility set.

In March 1979, while the author was Visiting Professor at Northwestern University, Brown visited NWU to give a talk on a more elaborate version of his investigation with Heal. Flattered by his mention of my proof of the existence of equilibrium, and my example showing the possibility of an inefficient equilibrium in the model of increasing returns to scale, I returned my attention to the subject. The outcome can be seen in the preceding lines.

My indebtedness to Don Brown is due not only to his constant insistence to publish my results, but also to the numerous discussions on this subject and others related to it, especially those related with problems of non-convex mathematical programming, subject to which I hope to return in an investigation on Lagrange multipliers.

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